Short Note

Laboratory results for the features of body wave propagation in a transversely isotropic medium

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INTRODUCTION

Much of the earth’s crust appears to have some degree of elastic anisotropy (Crampin, 1981; Crampin and Lovell, 1991; Helbig, 1993). The phenomena of elastic wave propagation in anisotropic media are more complex than those in isotropic media. Shear-wave propagation in an orthorhombic physical model is most complex when the direction of the wave is close to the neighborhood of the cusp on the group velocity surfaces (Brown et al., 1991). The first identification of singularities in wave propagation through sedimentary basins occurred in the examination of shear-wave splitting in multi-offset VSPs at a borehole site in the Paris Basin (Bush and Crampin, 1991), where large variations in shear-wave polarizations in propagation directions close to point singularities were observed. Computation of synthetic seismograms for layer sequences showed that the shear-wave polarizations and amplitudes were irregular near point singularities (Crampin, 1991).
Phase and group velocity are identical in homogeneous non-dispersive isotropic media. However, in anisotropic media, the direction of the two velocities is not necessarily the same, but the projection of the group velocity vector on the wave normal is equal to the phase velocity (Helbig, 1984). In reflection and VSP seismology, the source and receiver are usually assumed to be at a single point, so that the group velocity of the seismic wave is measured. The elastic properties of layered rocks are often measured using the pulse through-transmission technique, which measures the phase velocity of ultrasonic waves propagating in rock samples (Dellinger and Vernik, 1994). Therefore, the kind of velocity being measured is particularly important. However, it is difficult to discriminate between the waves which propagate at either the phase or group velocities. It is also difficult to observe the phase and group velocity at the same time by one seismic or ultrasonic method in an anisotropic medium.

For propagation directions near the cusps, the group velocity and polarization of seismic waves may undergo rapid variations for small changes in direction (Crampin, 1981; Helbig and Schoenberg, 1987; Crampin, 1991). These effects depend on frequency, source polarization, and degree of anisotropy (Crampin, 1991). Therefore, the features of seismic waves propagating through anisotropic rocks can not be easily observed or quantified from field data. Chang et al. (1994) clearly
demonstrated the phenomena of shear-wave splitting in a transverse isotropic (TI) medium, and in their laboratory study the group velocity surface of the qP- and qS-waves fit the theoretical ones well. However, because the degree of apparent anisotropy of the investigated medium was not strong enough, the phenomena of rapid variations of the group velocity and polarization of waves for directions approaching the neighborhood of cusps on the group velocity surface were not successfully observed.

In this current study, a well-controlled laboratory experiment was performed to observe body waves propagating with both phase and group velocities in a transverse isotropic (TI) medium, particularly for propagation directions near cusps.

**EXPERIMENTS**

An experimental set up similar to that used by Chang et al. (1994) was employed to measure the transit times of body waves as a function of polarization and propagation direction in a TI medium. The experimental error of the velocity was 1%. Phenolite composed of layers of paper bonded with phenolic resin, with density $\rho = 1.4 \times 10^3$ kg/m$^3$, was used as the modeling material. This paper-like material is transversely isotropic, and has no preferred direction of propagation.

Four kinds of transducers and two kinds of specimens (cubes and ellipsoid) were used in these experiments to measure the phase and group velocities (Table 1). A
pair of large size and low frequency transducers, and ten small size specimens (cubes) were used to measure the phase velocities while the transducers were in plane contact with the specimens (Panametrics A101R for the qP-waves and Ultran SWC50-1 for the qS-waves). The ten phenolite cubes with the same dimensions, $3\text{cm} \times 3\text{cm} \times 3\text{cm}$, but with different orientations ($\theta$) of the symmetry axes with respect to the measuring direction were used. The symmetry axis of the phenolite cubes was angularly tilted at $5^\circ$ increments from the in-line direction to the perpendicular observation direction. The ratio of the propagation distance of the wave to the diameter of the transducer used in the experiments was less than 3. The pulse-transmission experiments measured phase velocities instead of group velocities since the separation between source and receiver was three times greater than the transducer width (Dellinger and Vernik, 1994). In order to measure group velocities, a pair of small size but high frequency transducers were diametrically mounted onto an elliptical specimen (Panametrics A133S for the qP-waves and Ultran SWC18-5 for the qS-waves). The lengths of the short and long axes of the phenolite ellipsoid were 3.75 cm and 8 cm, respectively. The symmetry axis for the phenolite ellipsoid was perpendicular to the long axis, and the plane plates of the transducers were tangent to the curved surface of the ellipsoid. Therefore, the transducers were in point contact with the ellipsoid. During the experiment, the shear-wave transducers are always lined up and
simultaneously rotated in step. When the polarization of the shear-wave was parallel
to the symmetry plane, the qSV-wave was observed, otherwise when the polarization
was orthogonal to the symmetry plane, the SH-wave was observed. Therefore,
quadrantal observations of body wave propagation could be obtained through the
transmission experiment from $\theta=0^\circ$ to $\theta=90^\circ$ at angular intervals by changing the
cubes or rotating the ellipsoid.

**RESULTS**

The theoretical phase velocities of the body wave in a homogeneous TI media
can be derived from the equations of motion (Daley and Hron, 1977; Thomsen, 1986).
The group velocity is not easily derived for a wave radiated from a point source but
can be obtained from the phase velocity using a differential operator (Thomsen, 1986).
The physical properties of a TI medium can be described using five independent
elastic constants (Crampin, 1981; Helbig, 1993). The elastic constants normalized
by the density of the phenolite used in this study are $A_{11}=15.8$, $A_{13}=5.0$, $A_{33}=8.7$,
$A_{44}=2.2$, and $A_{66} = 4.5 \times 10^6$ m$^2$/s$^2$ from best fitting the observed data. The velocity
anisotropies ((Vmax-Vmin)/Vmax) of the phenolite model for qP- and qS-waves are
25% and 29%, respectively.

The traces of qSV-waves propagating with group velocities from $\theta=0^\circ$ to $90^\circ$
at an angular interval of 5° are shown in Figure 1. The amplitude scale for each trace in
the figure is arbitrary and the line with cusps is the theoretical arrival times of the qSV-waves propagating with group velocities. The first arrival times of the qSV-waves fit very well the theoretical arrival times for the qSV-waves propagating with group velocities. The velocity increases rapidly near the cusps and reaches a maximum at the cuspidal directions, confirming that the events we observed are the qSV-waves propagating with group velocities.

The velocity sections of the experimental results (symbols) and theoretical velocities (lines) for the qP-, SH- and qSV-waves are shown in Figures 2, 3 and 4, respectively. The ordinate is the velocity when the wave direction is perpendicular to the symmetry plane, and the abscissa is the velocity when the wave direction is parallel to the symmetry plane. The difference between the body waves propagating at the phase and group velocities can be appreciated; the observed phase and group velocities of the qP- and qS-waves fit the theoretical velocity curves very well.

**DISCUSSION AND CONCLUSIONS**

The measurement error of the amplitude on the elliptical specimen was very large and the scales of the amplitudes of the traces are arbitrary in Figure 1. Assuming that the level of background noise is the same throughout the measurement time, then the magnitudes of the amplitude of the signal can be estimated by comparison with the level of background noise. Along the cuspidal directions
(θ=40° and 45°), our laboratory data show that the magnitudes of the amplitudes of the qSV-waves clearly become large compared to the background noise. The qSV-waves may reach the cusps from different ray paths in succession. Therefore, the signals along one ray path will suffer (constructive or destructive) interference from signals from neighboring ray paths, where the polarizations and amplitudes may be different. For normally polarized TI media, the direction of the polarization of the qSV-waves is not necessary perpendicular to the ray direction, but the deviation of the polarizations from orthogonality to the ray direction is not large (Helbig and Schoenberg, 1987). The differences of the arrival time and deviation of the polarizations between different ray paths are not large in this study; therefore the signals can be added constructively at θ=40° and 45°.

The direction θ=35° is close to the cusps. Since the contact area joining the source (and receiver) and specimen is not a single point but a very small area, the stress and strain are continuous in the material, and the amplitude of the traces received at θ=40° and 45° are large, thus the particle motion at θ=35° will be affected by the particles at the cusps. Therefore, the precursor event with small amplitude that arrives before the qSV-wave propagating at the group velocity is observed at θ=35°.

The phase and group velocities are quite different in a strong TI medium, and are verified by this experiment. Therefore, a geophysicist must be careful in using the
wave speed of the anisotropic media to construct the rock properties. The variations in polarizations, amplitudes and velocities of the body wave propagating in a TI medium were observed in this study. The variations for the qSV-waves propagating with group velocities are very large in the vicinity of the cusps. Thus, if they are not correctly processed and interpreted, the results will deviate from the actual situation.

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We wish to thank Prof. K. Helbig, Prof. C. McCann, and anonymous reviewers for providing very constructive suggestions in the revision of this paper. This research was financially supported by the National Science Council under the contract no. NSC 85-2111-M-023-001 and NSC 87-2116-M-194-003.

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FIGURE CAPTIONS

**Figure 1** Observed result of qSV-waves propagating with group velocities from the symmetry axis for the phenolite ellipsoid being parallel \((\theta=0^\circ)\) to perpendicular \((\theta=90^\circ)\) to the observation direction. Following the theoretical arrivals (solid line with cusps), the so-called “cusps” of the qSV-wave can be easily identified.

**Figure 2** The qP-waves velocity surfaces of the modeling material. The symbols indicate the experimental results and lines are the theoretical curves. The ordinate is the velocity when the wave direction is perpendicular to the symmetry plane, and the abscissa is the velocity when the wave direction is parallel to symmetry plane.

**Figure 3** The SH-waves velocity surfaces of the modeling material. The symbols indicate the experimental results and the lines are the theoretical curves.

**Figure 4** The qSV-waves velocity surfaces of the modeling material. The symbols indicate the experimental results and the lines are the theoretical curves.
Table 1 Transducers and specimens used in the experiment to observe the body wave propagating with the phase and group velocities.

<table>
<thead>
<tr>
<th>Transducers</th>
<th>Type</th>
<th>Model</th>
<th>Diameter</th>
<th>Frequency</th>
<th>Propagation distance</th>
<th>Velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>P-wave</td>
<td>Panametrics</td>
<td>1 inch</td>
<td>0.5 MHz</td>
<td>3 cm (Cube)</td>
<td>Phase (plane contact)</td>
</tr>
<tr>
<td></td>
<td>P-wave</td>
<td>Panametrics</td>
<td>0.25 inch</td>
<td>2.25 MHz</td>
<td>7.5 cm (Ellipsoid)</td>
<td>Group (point contact)</td>
</tr>
<tr>
<td></td>
<td>S-wave</td>
<td>Ultran</td>
<td>0.5 inch</td>
<td>1 MHz</td>
<td>3 cm (Cube)</td>
<td>Phase (plane contact)</td>
</tr>
<tr>
<td></td>
<td>S-wave</td>
<td>Ultran</td>
<td>0.25 inch</td>
<td>5 MHz</td>
<td>7.5 cm (Ellipsoid)</td>
<td>Group (point contact)</td>
</tr>
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Figure 3
Figure 4